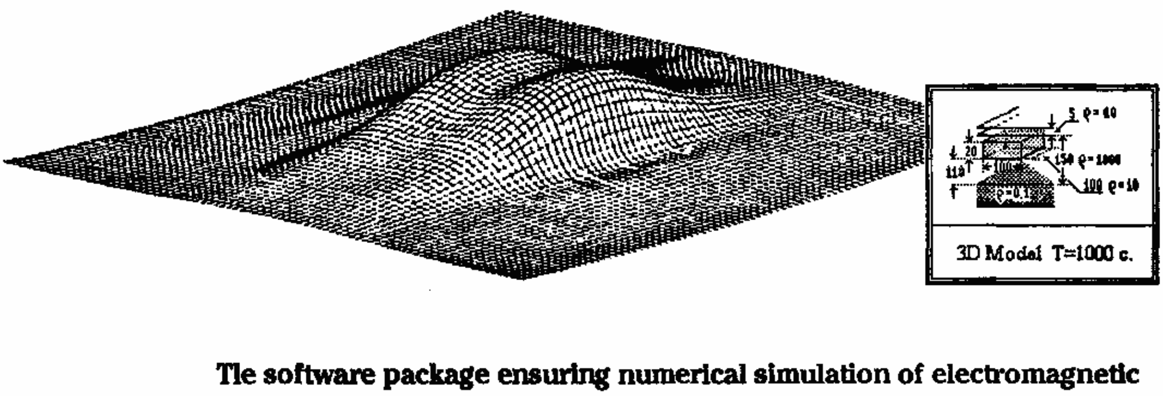
MADAM Maxwell

Geoelectricity

for Windows



Tle software package ensuring numerical simulation of electromagnetic fields in heterogeneous media

(Modelling Application of Decomposition Alternating Method)

1. Mathematical Statement of Problems

1.1. Model of the Medium and Field Sources

(a). Model of the Medium

The software package is developed for solving 1D, 2D and 3D problems that are of interest for structural, mining and deep-penetration survey. Each of the above methods uses specific versions of geoelectric section model. However, they share the stratified model of the enclosing medium whose electromagnetic properties depend on one spatial coordinate.

The algorithms use a rectangular Cartesian system of coordinates, in which the *xoy* plane corresponds to the earth/air interface. The z-axis is directed downwards. The horizontally homogeneous model of the medium is referred to as the normal section, while the fields formed by various sources in the normal section are termed normal fields. As a rule, it is assumed that this model bears on an unlimited-thickness bed offering uniform conductivity. In structural and mining problems, the bearing bed is usually represented by a crystalline foundation possessing a very low conductivity. In deep-penetration surveys, the bearing bed corresponds to the conductive layer of upper mantle, thus it has a sufficiently large conductivity. Air is assumed to be a poor conductor. In certain cases we shall treat air as an electric insulator (Fig. 1.1).

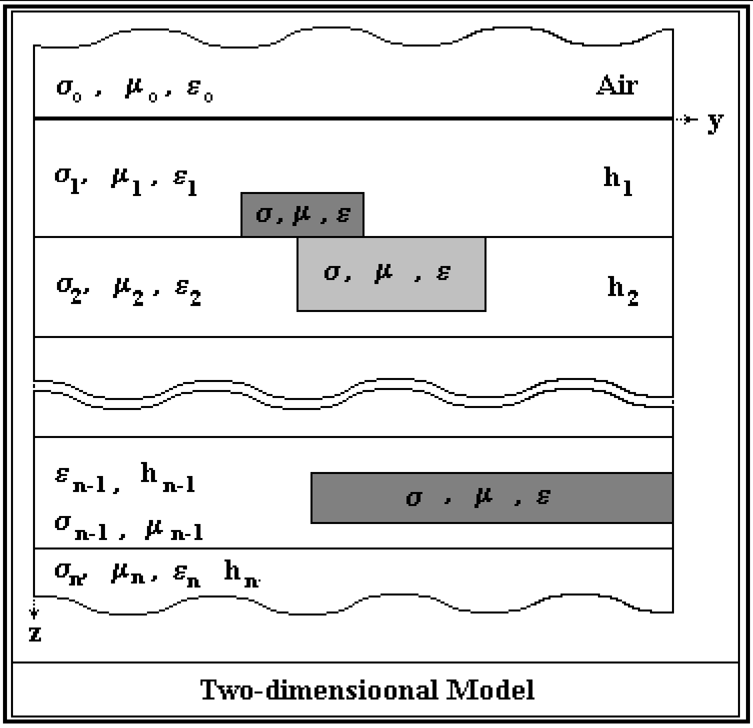


Fig. 1.1

Another assumption, used in 2D models, is that the medium properties remain invariable along x-axis.

Conductivity σ, magnetic permeability μ and dielectric permittivity *ε* are assumed to be piece wise-smooth functions. The boundaries where these functions have discontinuities shall also be deemed piece wise smooth.

Let's consider generally conductivity of the media as dependent not only from spatial coordinates of a point $\bar{x} = (x_1, x_2, x_3) \equiv (x, y, z)$, but also from circular frequency *ω*. For example, frequency dispersion of conductivity of the media caused by polarizing processes, it is accepted polarization effect by the formula Cole-Cole

$$\sigma(\bar{x},\omega) = \sigma_\infty(\bar{x})\left[1 - \frac{\eta}{1+(-i\omega\tau)^c}\right], \ \eta := \frac{\sigma_\infty - \sigma_0}{\sigma_\infty}, \ \sigma_\infty := \lim_{\omega\to\infty}\sigma(\bar{x},\omega), \ \sigma_0 := \sigma(\bar{x},0).$$

Here c.- constant; η - polarization of the media, τ - constant of the polarization time.

The "Normal[1] and the "Background[1] Medium Models

In geoelectrical studies, a model shaped as a horizontally stratified medium is usually referred to as the normal geoelectrical section, while the electromagnetic fields corresponding to the *normal model* are called *normal fields.*

Let us generalize on the concept of normal section and introduce the notion of a background geoelectric section and background fields. A background model will be chosen depending on what models render themselves to solution by the moment the current model is analyzed.

A background geoelectrical section or a *background* medium *model* will be understood as such a relatively simple model of the medium incorporating the heterogeneity under consideration, for which there exists a numerical or analytical solution of the direct problem in a multitude of points required for the solution of the complete problem.

With respect to the class of 1D problems, the background medium model may be represented by a uniform half-space or other "simpler" 1D models.

With respect to the class of 2D problems, this function may be performed by 1D models or 2D problems that are "simpler" than the one under consideration.

For 3D models, background medium model may be designed as 1D, 2D or "simpler" 3D models. Fields corresponding to background models will be referred to as background fields.

Conditions at Junctions and Conditions in the Infinity

Primal problems of geoelectricity are usually solved in an unlimited domain. If the problems are to have unique solutions, the electromagnetic fields must meet the condition of radiation into infinity.

For stratified media, the principle of ultimate absorption is conveniently applied. Let us assume that $Re\ \kappa > 0$ and require that the solution diminish into infinity.

The tangential components of vectors $\mathbf{E}_\tau$ and $\mathbf{H}_\tau$ are known to run uninterrupted through borders of medium property discontinuity,

$$[\mathbf{E}_\tau] = 0; \quad [\mathbf{H}_\tau] = 0,$$

where **[*U*]** = **a** denotes a discontinuity, equal to **a**, of function ***U*** at the transition through the interface between media with different properties.

In models incorporating thin conductive sheets with a finite conductivity *S(x,y,z),* the continuity of the magnetic field tangential components is broken.

During transition through the sheet, the tangential component $\mathbf{E}_\tau$ t of vector **E** remains continuous, while $\mathbf{H}_\tau$ undergoes a break equal to the value **J** of the surface current. Union conditions on an any smooth surface which the thin sheet is tense , can be written down in the following kind

$$[\mathbf{H}_\tau]\|_{\partial\Omega^S} = \mathbf{n} \times [\mathbf{H}]\|_{\partial\Omega^S} = S\mathbf{E}_\tau = \mathbf{J}, \tag{1.1}$$

where **n** is a unit vector normal to the sheet surface and directed from its negative side to the positive one, and the symbol x stands for vector product.

(b). Field Sources

Only finite functions will be considered as field sources.

In deep-penetration problems, the field is induced by a flat uniform monochromatic wave falling upon the Earth's surface, or by powerful artificial sources of electromagnetic fields, such as magnetohydrodynamic generators (MGD generators) of power-supply lines.

Mining and structural methods of electrical exploration are usually concerned with fields induced by man-made current sources, such as electric or magnetic dipoles, finite-length lines, cables, finite-dimension loops, etc.

Problem solution requires that conditions of field excitation should be set. If the electromagnetic field behavior close to the point of source location is taken into consideration, Maxwell homogeneous equations may be used (disregarding extrinsic current density/).

The programs are applicable to computation in case of field excitation by arbitrary source, provided software for determination of the normal field in an arbitrary point of a plane (2D problems) or space (3D problems).

About dimension of direct tasks the geoelectricity.

The dimension of a direct task d_t depends on dimension of model of the media d_m and source of a field d_s. These values are connected among themselves. For example, in a case 2D-models of media, applying suitable integral transformation, in the field of the images are possible manage dimension of a differential task. With reference to the cartesian system of coordinates the types of sources and models are represented in a fig. 1.1.2.

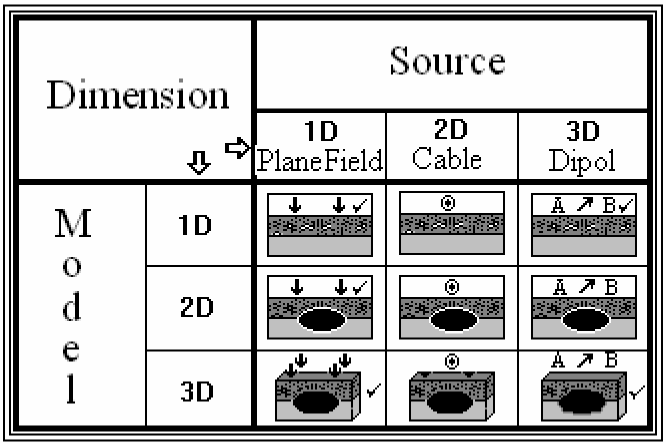


Рис. 1.1.2. Classifications of the media models and the fields sources.

In the consent with figure, further we shall speak about tasks, specifying their character of two parameters - dimension of model and dimension of a source (table 1.1):

Таблица 1.1

Задачи	Contents
1D+1D	1D media model and 1D field source
1D+2D	1D media model and 2D field source
1D+3D	1D media model and 3D field source
2D+1D	2D media model and 1D field source
2D+2D	2D media model and 2D field source
2D+3D	2D media model and 3D field source
3D+1D	3D media model and 1D field source
3D+2D	3D media model and 2D field source
3D+3D	3D media model and 3D field source

1.2. Differential Equations of Electromagnetic Fields

Theoretical basis the geoelectricity is the system of the Maxwell's equations

$$\begin{cases} rot\mathbf{H} = \sigma\mathbf{E} + \varepsilon\dfrac{\partial\mathbf{E}}{\partial t} + \mathbf{j}_s, \\ rot\mathbf{E} = \mu\dfrac{\partial\mathbf{H}}{\partial t}. \end{cases}$$

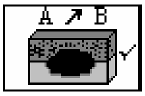
Here $\mathbf{E}(x,y,z,t)$ – vector of intensity of an electrical field, $\mathbf{H}(x,y,z,t)$ – vector of intensity of an magnetic field, $\mathbf{j}_s(x,y,z,t)$ – density of a current of outer sources of a field, $\sigma = \sigma(x,y,z,t)$, $\varepsilon = \varepsilon(x,y,z,t)$, $\mu = \mu(x,y,z,t)$.
Let's remind useful parities

$$\mathbf{D} = \varepsilon\mathbf{E},\ \mathbf{B} = \mu\mathbf{H},\ \mathbf{j} = \sigma\mathbf{E},$$

in witch $\mathbf{D}$ – vector of an electrical induction, $\mathbf{B}$ – vector of an magnetic induction, $\mathbf{j}$ – density of a current of conductivity. In the first Maxwell's equation the value $\varepsilon\,\partial\mathbf{E}/\partial t$ has dimension of conductivity and is called as a *current of displacement*.

As a rule, the values σ, ε and μ, also believe not time-dependent. However in this case not it is possible to coordinate all electromagnetic processes occurring in mountain rocks, having mainly by ionic conductivity, to results of calculations based on such simplification of model of media.

1.2.1. Fields in frequency domain



3D+3D problems. We shall assume, that the complex amplitudes a component of vectors of intensity electrical $\mathbf{E}$ and magnetic $\mathbf{H}$ of fields change under the law exp($-i\omega t$). Using system of units CI and standard designations, we shall write down system of the Maxwell's equations in the following kind:

$$\begin{cases} rot\mathbf{H} = \tilde{\sigma}\,\mathbf{E} + \mathbf{j}_S \\ rot\mathbf{E} = i\omega\mu\mathbf{H}, \\ div\mathbf{B} = 0, \\ div\mathbf{D} = p. \end{cases} \tag{1.2.1}$$

Here $\tilde{\sigma} = \sigma - i\omega\varepsilon$ - complex conductivity, $\mathbf{j}_S$ – outer field sources, p – density of volumetric chargesИз первого уравнения системы (1.2.1) следует

$$div(\tilde{\sigma}\mathbf{E}) = -div\mathbf{j}_S$$

Or and from secondary

$$div\,\mathbf{E} = -div\mathbf{j}_S - (E, grad\tilde{\sigma}),$$

and from secondary equation by this system we have

$$div\mathbf{B} = 0.$$

From (1.2.1) serial exception $\mathbf{E}$ and $\mathbf{H}$ receive

$$rot\,\mu^{-1}rot\mathbf{E} + k^2\mu^{-1}\mathbf{E} = i\omega\mathbf{j}_s$$

$$rot\tilde{\sigma}^{-1}rot\mathbf{H} + k^2\tilde{\sigma}^{-1}\mathbf{H} = rot\left[\mathbf{j}_s\tilde{\sigma}^{-1}\right] \tag{1.2.2}$$

where $k^2 = -i\omega\mu\tilde{\sigma}$. Further for simplification of record instead of $\tilde{\sigma}$ shall write σ.

The electromagnetic field in inhomogeneous media is convenient for representing as the sum of background $\mathbf{E}^n$, $\mathbf{H}^n$ and anomalous $\mathbf{E}^a$, $\mathbf{H}^a$ fields

$$\mathbf{H} = \mathbf{H}^n + \mathbf{H}^a,$$

$$\mathbf{E} = \mathbf{E}^n + \mathbf{E}^a.$$

Аналогично, σ и μ представим в виде сумм проводимости и магнитной проницаемости фоновой модели среды (σ^n, μ^n) и их аномальных составляющих (σ^a, μ^a):
Analogous, and we shall present as the sums of conductivity and magnetic permeability of background model of media (σ^n, μ^n) and their anomal components (σ^a, μ^a):

$$\sigma(x,y,z) = \sigma^n(x,y,z) + \sigma^a(x,y,z),$$

$$\mu(x,y,z) = \mu^n(x,y,z) + \mu^a(x,y,z).$$

Уравнения Максвелла для фоновых полей с теми же сторонними источниками поля $\mathbf{j}_s$, что и в (1.2.1) имеют вид:

The Maxwell's equations for background fields with same by outer sources of a field, as in (1.2.1) look like:

$$\begin{cases} rot\mathbf{H}^n = \sigma^n \mathbf{E}^n + \mathbf{j}_s, \\ rot\mathbf{E}^n = i\omega\mu^n \mathbf{H}^n. \end{cases} \tag{1.2.3}$$

Subtracting from the equations of system (1.2.1) appropriate equations of system (1.2.3), we shall receive

$$\begin{cases} rot\mathbf{H}^\alpha = \sigma\mathbf{E}^\alpha + \sigma^\alpha \mathbf{E}^n, \\ rot\mathbf{E}^\alpha = i\omega\mu\mathbf{H}^\alpha - i\omega\mu^\alpha \mathbf{H}^n. \end{cases} \tag{1.2.4}$$

In right parts of the equations include expressions $\mathbf{j_a} = \sigma^\alpha \mathbf{E}^n$, $\mathbf{j_b} = -i\omega\mu^\alpha \mathbf{H}^n$. They are superfluous electrical and magnetic currents being sources of anomalous electromagnetic fields. Solving system (1.2.4) rather **E** and **H**, we shall receive

$$rot\frac{1}{\mu}rot\mathbf{E}^a - i\omega\sigma\mathbf{E}^a = i\omega\sigma^a\mathbf{E}^n + i\omega rot\left[\frac{1}{\mu}\mu^a\mathbf{H}^n\right], \tag{1.2.5}$$

$$\mathrm{rot}\frac{1}{\sigma}\mathrm{rot}\mathbf{H}^a - i\omega\mu\mathbf{H}^a = i\omega\mu^a\mathbf{H}^n + \mathrm{rot}\left[\frac{1}{\sigma}\sigma^a\mathbf{E}^n\right]. \tag{1.2.6}$$

We shall enter designations

$$L := rot\frac{1}{\eta}rot, \quad L_E := rot\frac{1}{\mu}rot, \quad L_H := rot\frac{1}{\sigma}rot$$

$$\mathbf{f} := \begin{cases} \mathbf{f}_E := i\omega\sigma^a\mathbf{E}^n + i\omega\cdot rot\left[\mu^{-1}\mu^a\mathbf{H}^n\right], \\ \mathbf{f}_H := i\omega\mu^a\mathbf{H}^n + rot\left[\sigma^{-1}\sigma^a\mathbf{E}^n\right]. \end{cases} \tag{1.2.7}$$

The equations (1.2.5), (1.2.6) have identical structure, we therefore can write down them in the following kind:

$$L\,\mathbf{u} + \frac{k^2}{\eta}\mathbf{u} = \mathbf{f}, \qquad k^2 := -i\omega\mu\sigma. \tag{1.2.8}$$

From (1.2.8) the equations for complete and anomalous electromagnetic fields turn out if to make substitutions in the consent with the table 1.2.

Thus, at account of anomalous fields the role of sources of a field is played by superfluous currents determined by the formula (1.2.7), concentrated in the location inhomogeneities of model of media distinguished from properties of background model and raised by a background field of a concrete source. In this case computing circuits invariant in relation to a type of activators of a field. For the solution of a problem it is necessary to be able to calculate a background fields in area a $\sigma^a \neq 0$, $\mu^a \neq 0$, and also in those points, in which it is necessary to find complete fields. By search and investigation of petroleum fields it is possible to put $\mu^a = 0$.

Table 1.2

			$rot\frac{1}{\eta}rot\mathbf{u}+\frac{k^2}{\eta}\mathbf{u}=\mathbf{f},\quad k^2:=-i\omega\mu\sigma.$
u	L	η	**f**
E	L_E	μ	$i\omega\,\mathbf{j}_s$
$\mathbf{E}^a$	L_E	μ	$i\omega\cdot rot\left[\mu^{-1}(\mu-\mu^n)\mathbf{H}^n\right]+i\omega(\sigma-\sigma^n)\mathbf{E}^n$
H	L_H	σ	$rot\left[\mathbf{j}_s\sigma^{-1}\right]$
$\mathbf{H}^a$	L_H	σ	$rot\left[\sigma^{-1}(\sigma-\sigma^n)\mathbf{E}^n\right]+i\omega\cdot(\mu-\mu^n)\mathbf{H}^n$
Differential equations of EM Fields			

Then the formula (1.2.7) will have obvious simplification. In particular

$$L_E\mathbf{E}=\frac{1}{\mu}rotrot\mathbf{E}=\frac{1}{\mu}\left(graddiv\mathbf{E}-\Delta\mathbf{E}\right) \qquad (1.2.11)$$

In this case last formula can be presented in a little bit other kind. Influencing by the operator div on both parts of the first equation of system (1.2.4), we shall receive

$$\mathrm{div}\left(\sigma\mathbf{E}^a\right)+\mathrm{div}\left(\sigma^a\mathbf{E}^n\right)=0\,.$$

As

$$div\left(\sigma\mathbf{E}^a\right)=\left(grad\,\sigma,\mathbf{E}^a\right)+\sigma div\mathbf{E}^a\,,$$

we have

$$div\mathbf{E}=-\frac{1}{\sigma}\left(grad\,\sigma,\mathbf{E}\right)+div\left(\sigma^a\mathbf{E}^n\right).$$

Hence,

$$graddiv\mathbf{E}=grad\left[-\frac{1}{\sigma}\left(grad\,\sigma,\mathbf{E}^a\right)+div\left(\sigma^a\mathbf{E}^n\right)\right]=$$

$$=-grad\left[\sigma^{-1}\left(grad\,\sigma,\mathbf{E}^a\right)\right]+grad\left[\frac{1}{\sigma}div\left(\sigma^a\mathbf{E}^n\right)\right].$$

And equation (1.2.8) with reference to an electrical field accepts a kind :

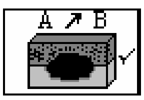
$$\nabla^2\mathbf{E}^a-\nabla\left[\frac{\nabla\sigma}{\sigma}\mathbf{E}^a\right]+k^2\mathbf{E}^a=₣ \qquad (1.2.12)$$

where

$$₣=\mathbf{f}_E+\nabla\frac{\nabla\left(\sigma^a\mathbf{E}^n\right)}{\sigma}\quad.$$

Here ∇ - Hamilton's differential operator.

1.2.2. Transient EM Fields



3D+3D problems.

In the consent with the Maxwell's equations, written down in neglect of displacement currents

$$\begin{cases}\mathrm{rot}\,\mathbf{H}=\sigma\mathbf{E}+\mathbf{j}_s\,,\\ \mathrm{rot}\,\mathbf{E}=-\mu\dfrac{\partial\mathbf{H}}{\partial t}\,,\end{cases}$$

E H fields satisfy to the equation

$$rot\frac{1}{\eta}rot\mathbf{V}+q\frac{\partial\mathbf{V}}{\partial t}=\mathbf{f} \qquad (1.2.13)$$

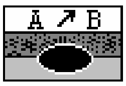
Table 1.4.

V	L	η	q	f
E	L_E	μ	σ	$-\dfrac{\partial\mathbf{j}_s}{\partial t}$
$\mathbf{E}^a$	L_E	μ	σ	$-(\sigma-\sigma_n)\dfrac{\partial\mathbf{E}^n}{\partial t}-rot\left[\mu^{-1}(\mu_n-\mu)\dfrac{\partial\mathbf{H}^n}{\partial t}\right]$
H	L_H	σ	μ	$rot\left(\mathbf{j}_s\sigma^{-1}\right)$
$\mathbf{H}^a$	L_H	σ	μ	$-(\mu_n-\mu)\dfrac{\partial\mathbf{H}^n}{\partial t}-rot\left[\sigma^{-1}(\sigma_n-\sigma)\dfrac{\partial\mathbf{E}^n}{\partial t}\right]$

The equations for complete (***E*** and ***H***) and anomalous ($\boldsymbol{E}^{\mathbf{a}}$ and $\boldsymbol{H}^{\mathbf{a}}$) fields turn out by substitution in (1.2.13) instead of L, V, q and f of functions given in the table 1.4.

For selection of the unique solution of the equation of a kind (1.2.13) it is necessary, except for boundary conditions or conditions on infinity, to set at the initial moment of time $t=t_0$ of values of fields in all points of region, in which the solution is searched

$$V(P,t)\big|_{t=t_0}=\varphi(P), P\in\mathbb{R}^3.$$



1.3. Variational Problem Statement

Variational approaches to the solution of geoelectricity problems are discussed in V.V. Nikolsky's monograph [Nikolsky, 1967] gives a treatment of these issues with respect to electrodynamics of hollow systems. The variational approach to the solution of problems the geoelectricity is considered in papers [Coggon,1971; Rodi,1976].

If the variability of the magnetic permeability and dielectric permittivity is taken into consideration, the variational functional assumes the following form:

1.3.1. . Functionals for EM Fields in frequency domain

A. **Common case** *(3-D - functionals)*.

We shall allocate the finite domain $\Omega\subset\mathbb{R}^3$ with piece-smooth border. Scalar product for functions from complex Hilbert space define as follows

$$\langle\mathbf{U},\mathbf{W}\rangle_\Omega=\int_\Omega\mathbf{U}\mathbf{W}^*d\tau\ ,$$

where the vector $\mathbf{W}^*$ conjugate to vector **W**. Scalar product on a surface similarly enters by

$$\langle\mathbf{U},\mathbf{W}\rangle_{\partial\Omega}=\int_{\partial\Omega}\mathbf{U}\mathbf{W}^*ds$$

We shall designate external border of domain Ω by a symbol $\partial\Omega_e$, and internal borders of break of properties of the madia (coefficients k и η) by symbols $\partial\Omega_i$ (i =1,...,n).

Let take a problem

$$\begin{cases} L\ \mathbf{v}+k^2\eta^{-1}\mathbf{v}=\mathbf{f}, \\ [\mathbf{v}_\tau]\big|_{\partial\Omega_i}=0,\ \left[\left(\eta^{-1}rot\mathbf{v}\right)_\tau\right]\big|_{\partial\Omega_i}=0, \\ l\mathbf{v}\big|_{\partial\Omega_e}=\mathbf{u}, \end{cases} \qquad (1.3.1)$$

where l - some linear differential operator, $\mathbf{f} \in L^2(\Omega)$, $\mathbf{v} \in \Omega_L$.
Variational functional assumes the following form:

$$\mathrm{F}(\mathbf{U},\mathbf{V}) = \langle \eta^{-1} rot\mathbf{V}, rot\mathbf{U} \rangle + \langle k^2 \eta^{-1}\mathbf{V}, \mathbf{U} \rangle - \langle \mathbf{V}, \tilde{\mathbf{f}} \rangle - \langle \mathbf{f}, \mathbf{U} \rangle \qquad (1.3.2)$$

In the right part of equation (1.3.2), the expression (1.3.2) denotes a scalar product in a complex functional space $L_2(\Omega)$ in the domain Ω to which the variational problem statement refers.

By calculating the first variation δF*,* for example, for argument **V**, it is easy to verify that formula (1.2) is the Eulerian equation for functional (1.3.2)

$$\delta \mathrm{F}_u = \langle rot \eta^{-1} rot\mathbf{V} + k^2 \eta^{-1}\mathbf{V} - \mathbf{f}, \mathbf{V} \rangle = 0. \qquad (1.3.3)$$

Moreover, at the stationary value of integral (1.3.2), reached at function U_s, the natural boundary-value conditions are met:

1. at the external border $\partial\Omega$ Q of domain Ω:

$$\mathbf{n} \times \left(\eta^{-1} rot\mathbf{U}_s\right)\Big|_{\partial\Omega_e} = 0 \qquad (1.3.4)$$

2. and at the internal borders, where medium properties change, conditions of interface

$$\left[(\mathbf{U}_s)_\tau\right]\Big|_{\partial\Omega_i} = 0, \quad \left[\left(\eta^{-1} rot\mathbf{U}_s\right)_\tau\right]\Big|_{\partial\Omega_i} = 0. \qquad (1.3.5)$$

where **n** is a unit vector normal to surface $\partial\Omega$; **U = E | H**.
Thus, the conditions of continuity of tangential components of vectors E and H at borders of medium properties discontinuities are natural border-value conditions for functional (1.3.2).
Functional (1.3.2) must be modified to suit models incorporating thin current-conductive sheets (S sheets). In order to convert the border-value conditions on the sheet into a natural one, sum must be added to them so as to eliminate integrals ∂S over sheet surfaces. For functional (1.3.2) this can be achieved by means of the following formula:

$$\mathrm{F}_s(\mathbf{U},\mathbf{V}) = \mathrm{F}(\mathbf{U},\mathbf{V}) + i\omega \sum_m \langle S_m \mathbf{U}, \mathbf{V} \rangle \qquad (1.3.6)$$

where ∂S is the conductivity of m-th sheet, while integrals under the summation symbol are taken over sheet surfaces ∂S. Equation (1.3.5) is valid when **U** = **E**

2D functionals.

Let vector **V** has one distinct from zero to a component, dependent from two spatial variable y, z: **V** = (V(y,z), 0,0). Let's receive expressions for функционалов, and also differential equations (equation Эйлера), regional conditions and conditions of interfaces ensuring them стационарность. Let properties of the media do not vary on coordinate x. Then the function V will depend only from y and z, if the orientation of sources of a field is properly coordinated with 2D model of media. Let's assume, that 2D domain *D* has piece-smooth border *δD* and piece-smooth internal borders of the unit of subdomains with various electrical and magnetic properties D. In 2D case functional (1.3.2) after simple transformations can be written down in the following kind.

Let $\mathbf{U} = (U(y,z),0,0)$, $\mathbf{V} = (V(y,z),0,0)$, $\mathbf{f} = (f(y,z),0,0)$. For 2D-problems by analogue of formula (1.3.2) is

$$\mathrm{F}_2(U,V) = \int_D \left\{ \frac{1}{\eta}\left[\frac{\partial U}{\partial y}\frac{\partial V^*}{\partial y} + \frac{\partial U}{\partial z}\frac{\partial V^*}{\partial z} + k^2 U V^* \right] + U f^* + f V^* \right\} ds, \qquad (1.3.6)$$

The presence in model of thin S-sheets is taken into account by addition to last formula integrals by sheets surfaces δS_m

$$\mathrm{F}_{2s}(U,V) = \mathrm{F}_2(U,V) + i\omega \sum_m \langle S_m V, U \rangle \qquad (1.3.7)$$

Formula (1.3.7) can be used in E- polarization case only.

Functionals $F_2(U,V)$, $F_{2s}(U,V)$ becomes stationarity on functions, which are the solution of a problem

$$\begin{cases} \frac{\partial}{\partial y}\left(\frac{1}{\eta}\right)\frac{\partial U}{\partial y}+\frac{\partial}{\partial z}\left(\frac{1}{\eta}\right)\frac{\partial U}{\partial z}-\frac{k^2}{\eta}U=-f, \\ \left(\frac{1}{\eta}\frac{\partial U}{\partial n}\right)_{\partial D_e}=0, \\ \left[\mathbf{V}_x\right]_{\partial D_i}=0, \left(\frac{1}{\eta}\frac{\partial \mathbf{V}_x}{\partial n}\right)_{\partial D_i}=0, \end{cases} \tag{1.4.8}$$

Here

$$\left[\mathbf{V}_x\right]_{\partial D_i}=0, \left(\frac{1}{\eta}\frac{\partial \mathbf{V}_x}{\partial n}\right)_{\partial D_i}=0$$

natural boundary conditions on the internal boundaries ∂D_i and

$$\left(\frac{1}{\eta}\frac{\partial U}{\partial n}\right)_{\partial D_e}=0$$

natural boundary conditions at the external border D_e of domain D.
The private cases.
1. H-polarisation: U(y. z) = H_x (y, z). $\eta=\sigma$.
2. E-polarisation: U (y, z) = E_x (y, z), $\eta=\mu$.

1.3.2. Functionals for transient EM Fields in time domain.

At construction functionals for non-stationary problems the geoelectricity expediently derivative on time from required functions to consider as the given functions of spatial coordinates at some fixed moment of time *t* [Streng, Fix, 1977; Сегерленд, 1979]. Let to intensity of an electrical or magnetic field there corresponds a vector $\mathbf{U}$ having real components

1) 3-D - functionals.

Let's consider a functional for model of media containing S-sheet.

$$G(\mathbf{U})=\int_{\Omega}\left[\eta^{-1}(rot\mathbf{U})^2+2q\mathbf{U}\frac{\partial \mathbf{U}}{\partial t}-2\mathbf{fU}d\Omega\right]+2\sum_{j=1}^{m}\int_{\partial\Omega_j^s}S_j\mathbf{U}_\tau\frac{\partial \mathbf{U}_\tau}{\partial t}ds, \tag{1.4.9}$$

Here $\partial\Omega_j^S$ – surfaces, on which are tense of a S- sheets, $\mathbf{U}_\tau$ – vectors laying in tangential of a plane to a surface of a sheet such, that $\mathbf{U}=\mathbf{U}_\tau+\mathbf{U}_n$, $\mathbf{U}_n$ - vector, perpendicular to $\partial\Omega_j^S$.
For the first variation δG by means of the Gauss-theorem we shall receive expression

$$\delta G=\int_{\Omega}\left[rot\eta^{-1}rot\mathbf{U}+q\frac{\partial \mathbf{U}}{\partial t}-\mathbf{f}\right]\mathbf{W}d\tau+\int_{\partial\Omega^e}\left[\eta^{-1}rot\mathbf{U}\times\mathbf{W}\right]d\mathbf{s}.$$

where a vector $\mathbf{W}$ - variation of a vector $\mathbf{U}$.

The variation δG is equal to zero, if the vector - function $\mathbf{U}$ satisfies to the equation (1.2.13), to conditions of interface of a kind (1.3.5) and the regional condition of a kind (1.3.4) is carried out natural boundary conditions of kind .

2. Decomposition Alternating Method of Numerical Solution of Direct Problems of Geoelectricity

Decomposition methods are based on breaking down the initial problem into a number of simpler subproblems and individually solving each of the latter. The general solution is recomposed, as a rule, by an iterative process which serves to coordinate the subproblem solutions. Here belongs the Decomposition Alternating Method (DAM) which we develop as a generalization of the Schwartz alternating method over scalar and vector problems of geoelectricity (Yudin M.N.,1981; Vanian L.L.; Debabov A.S.,Yudin M.N.,1984; Yudin M.N.,1985;Yudin M.N.,Yudin V.M.,2004).

Convergence of the Schwartz method is analyzed in studies by S.L. Sobolev (1936) (equations of the theory of elasticity) and S.G. Mikhlin (1952) (for Dirichlet problems with positively determined elliptic differential operators with variable coefficients). Issues of convergence of the Decomposition Altemating Method as applied to geoelectricity problems were discussed in the dissertation by M.N. Yudin (1985).

In geoelectrical studies the algorithm may find application mainly in two interconnected aspects.

1. Proceeding from known solutions for comparatively simple areas, on their basis and bycombining them, it is possible to obtain solutions for domains with significantly morecomplicated structures.
2. Combination of problem solution by various methods in domains with non-empty intersectionsallows one to employ the methods most efficient for the kind of domain in question.

Taking the first way, one can obtain, for example, solution of a 2D problem for a geophysical profile, generally speaking, of an arbitrarily large length, which is especially important for interpretation of results of electric profile exploration work. In addition, the assumption of the local nature of 2D or 3D heterogeneity, which is extensively used in numerical solutions of inner border-value problems of geoelectricity, looses some of its limiting effect.

The other way of using the Decomposition Alternating Method consists in ensuring consistency of solutions of inner boundary-value problems, obtained by the use of such universal techniques as the finite elements method or finite difference method, with analytical solutions of outer boundary-value problems, found by spectral techniques.

In DAM, relations between partial problems are established through the boundary conditions, which provides significant computational advantages as compared to compositional methods, since the matrix structure of difference equations remains unaltered.

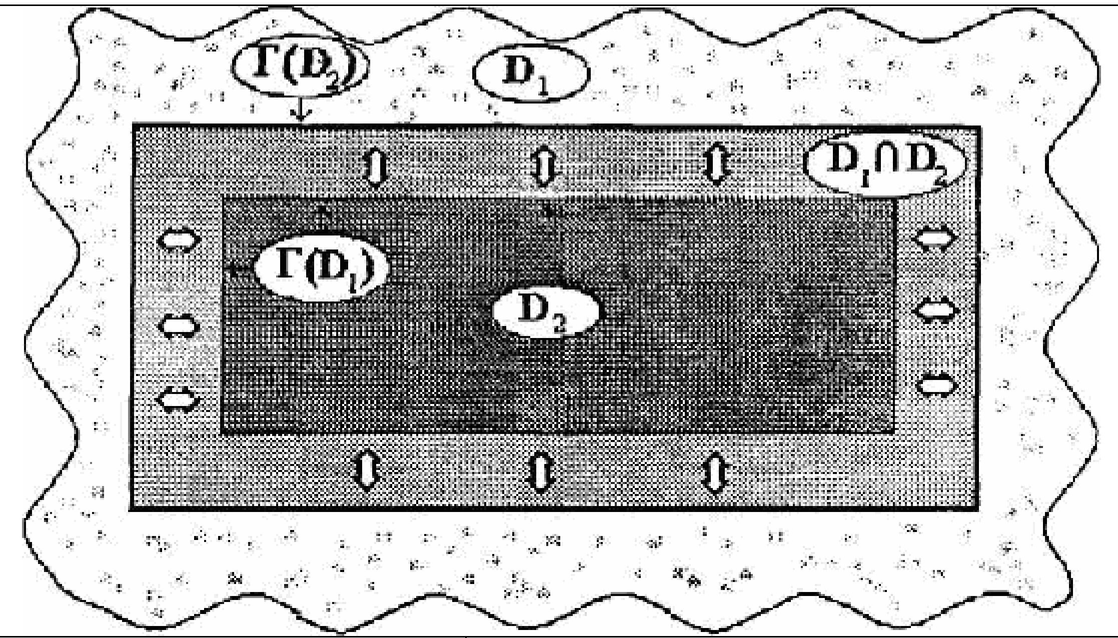
2.1. Classification of Decomposition Levels

Global Decomposition

A Decomposition Altemating Method which requires sequential independent solution of the inner and outer boundary-value problems will be referred to as the Global DAM (GDAM) or the Zero-Level DAM (DAM-0) (Fig. 2.1).

Secondary Decomposition

DAM that makes use of secondary decomposition in the process of inner boundary-value problem solution will be termed Local DAM (LDAM) or 1-st Level DAM. The larger the level number I (l>0), the closer the secondary decomposition is to the usual digitization used in numerical problem solution by the difference methods.



Exterioir infinite Domain D_1
Interior finite Domain D_2
Overlapping Domain D $D_1 \cap D_2$

Fig. 2.1 Global decomposition.

First Level Decomposition. When carrying out numerical simulation of electromagnetic fields and dealing with real-life wide-spread geoelectrical sections, it may prove impossible to perform simultaneous calculations for the entire medium model with a maximum number of nodes. Attempts to solve numerically even comparatively simple models demonstrate quite clearly that computer RAM is overloaded. DAM-based model decomposition and reduction of a complicated problem to a number of simpler problems which can be solved independently allows the situation to be notably improved and gives grounds to believe that real-life 3D problems will lend themselves to solution of the existing computers or computers of the near future.

The decomposition algorithm for solution of the inner boundary-value problem, the algorithm whose prime purpose is saving of the computer RAM, will be referred to as the First Level Decomposition Alternating Method (DAM-1).

DAM-1 removes practically all limitations on model size and allows electromagnetic fields to be simulated for wide-spread geoelectrical sections. Boundary values on the upper and lower borders of the grid have to be corrected, in GDAM, for the entire model at a time, on the basis of calculation results obtained for all autonomous blocks adjacent to these borders (Fig. 2.2).

Second Level Decomposition. The second and lower levels of decomposition aim at improving the efficiency of solving the direct problems in autonomous blocks operated by DAM-1, rather than at saving computer RAM.

Suppose a block proves to have such a structure that it can be broken into a number of parts which do not have to overlap and which allow the first boundary-value problem to be solved by efficient (in the ideal case, analytical) means. For example, if the block contains a certain number of horizontally uniform or vertically uniform fragments, then within each of the fragments the first boundary-value problem can be solved by the method of separation of variables, using the Fourier finite transformation applied to those coordinates along which the medium has constant properties.

Decomposition that proceeds separately from digitization, but is induced by distribution of heterogeneities in the geoelectrical section model and is performed with the view to increasing the efficiency of problem *solution within one first-level autonomous block, will be termed the second level decomposition (DAM-2) (Fig. 2.3).*

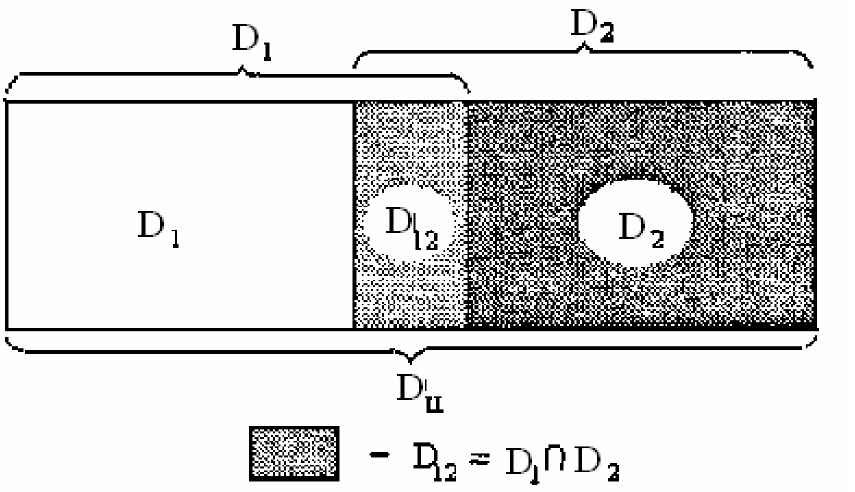
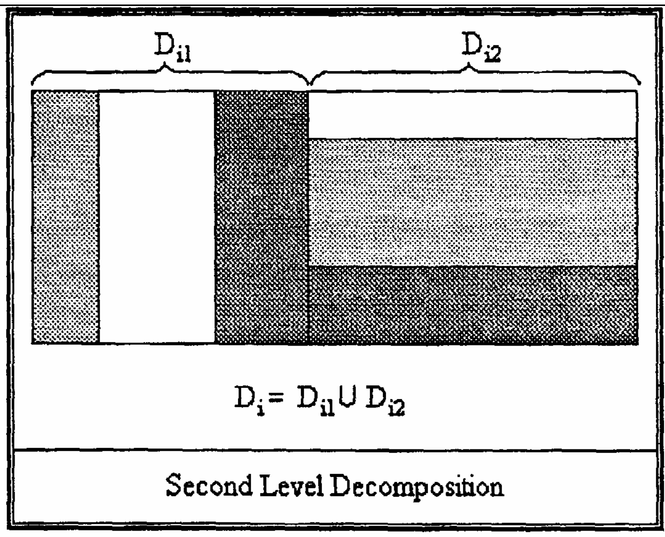


Fig. 2.2. First level decomposition



Fif. 2.3.

Solutions obtained in fragments with a common border (no overlap) may be sewed together by several methods.

1. The simplest of these boils down to "low-scale" Schwartz method application (Zavadsky.1982): the value of the respective component at the boundary is taken as equal to half-sum of the field values in points closest to the boundary and located at equal distances *h* on different sides from it. In this case the error of approximation to the boundary value has an asymptotic estimate not worse than O (h).

Let us illustrate this- by a simple example. In the case of E-polarization, the electric component of the MT field at the interface between air and uniform ground having a wave number κ is described by functions $l+k|z|$ (in the air) and (exp(-k|zj)) (in the earth). Finding the half-sum of the values of the exponent and the linear function, taken at identical sufficiently small distances from the air/ground interface:

$$1/2\left(1+kh+e^{-kz}\right) \approx 1+\left(kh\right)^2/4+...=1+O\left(h^2\right)$$

Hence, the error estimate here is equal to O *(h^2)*. Various aspects of the "small-scale" Schwartz method are discussed with respect to acoustic problems in (Zavadsky, 1982).

2. Sewing together of solutions obtained in non-intersecting domains may be done with the use of programs available for field simulation by difference methods. They are efficient for calculations performed only in narrow bands covering the fragment interface boundaries. If the band width is taken as equal to twice the grid pitch, concordance of solutions in the 2D case boils down to the solution of a set of equations with a three-diagonal coefficient matrix. Field values at the boundary are computed by the efficient sweep method algorithm.
Within the framework of DAM-2, in some (or all) fragments problems can be solved numerically (e.g., using the difference methods) by applying autonomous digitization to each partial domain. Such a need quite infrequently arises when problems are solved with respect to geoeiectrical sections featuring high contrast resistances (zones of mainland transition to ocean, islands in ocean, etc.).

Third-level decomposition has the same purpose as DAM-2, but differs from the latter in that it is closely connected with digitization.

Decomposition coordinated with digitalization and oriented at application of difference algorithms for the solution of inner boundary-value problems in regular overlapping minimum (elementary) domains whose boundaries coincide with grid layers, will be referred to as third-level decomposition (DAM-3).

A degenerate case of DAM-3 is represented by the Seidel method, when the elementary domains are clusters of the four finite elements (cells) surrounding each of the inner nodes of the grid. As each successive field value is determined in this node, it is assumed that the values at the boundary of the elementary domain are known (Fig. 2.4).
A wide spectrum of computation sequences may be obtained by selecting elementary local domains (ELD) of sufficiently large size and find autonomous solutions inside these domains, while known field values found in adjacent ELD are used as boundary conditions. In particular, if the field within ELD is computed by difference methods, the respective set of algebraic equations is easy to solve by direct methods, as the number of nodes is relatively small. This approach provides a way towards generalization of such well-known methods of calculation as the hop-scotch method or the alternating direction method (ADM) (Fig. 2.5).
Thus, the use of several hierarchical methods of decomposition enlarges the class of geoelectricity problems that lend themselves to solution of limited-power computers, and allows to develop new approaches to their analysis, as well as raise the efficiency of solving the traditional sets of difference equations, to which primal problems are reduced.

2.2. The Schwartz Method in 2D Problems

Solution of 2D problems involves two levels of decomposition (Fig.2.6). The global and the local ones.
Global decomposition
Global decomposition consists in the following. The plane is broken down into five subdomains (Fig.2.6), of which one is limited and incorporates the local heterogeneity under consideration, while the rest of the domains (two half-bands and two half-planes) are unbounded.

<table>
<tr><td colspan="3">1. AIR</td></tr>
<tr><td>3. LEFT
HALF BAND</td><td>5. GRID DOMAIN</td><td>4. RIGHT
HALF BAND</td></tr>
<tr><td colspan="3">BASIS</td></tr>
</table>

Fig. 2.6

Global Decomposition

Thus, global decomposition of a geoelectrical problem makes it necessary that two types of boundary-value problems should be solved:

- the outer boundary-value problems (in domains 1 - 4) and

- an inner boundary-value problem (in domain 5).

Solutions in the former two domains are found by calculations using well-known analytical techniques, while in domains 3 and 4 approximate asymptotic equations have to be used. In the simplest cases it is assumed that the normal derivative of field components is equal to zero over lateral boundaries of the grid domain. A solution found for the grid domain is, in conformity with the DAM algorithm, iteratively concorded with solutions for subdomains 1 - 4.

An algorithm has been developed that does not require overlap of subdomains for sewing solutions together.

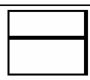
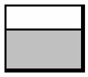
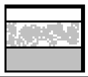
Algorithms for Solution of Outer Boundary Problems

Determination of Fields in Half-Planes (Domains 1 and 2)

Algorithms for determination of heterogeneous 2D and 3D fields in a horizontally stratified medium were derived by using the Fourier transform over spatial coordinates coinciding with strikes of the seams. In the domain of Fourier-images, analytical solutions have been found for the 1D boundary-value problem which allow spectral densities of electromagnetic fields in an arbitrary point.

The numeric realization of the algorithm has been based on the use of the Fast Fourier Transform (FFT) program.

For elementary models of horizontally uniform stratified medium, such as:

, (1)		unlimited thickness insulator
(2)		finite conductivity unlimited thickness stratum
(3)		limited thickness insulator overlying an ideal conductor

In the case of 2D fields, originals have been found which allow electrical and magnetic fields to be determined for an arbitrary point of the half-plane by computation of convolutions, without FFT application.

If a 2D heterogeneity is contained in a stratified medium possessing identical properties on the right hand and the left hand from the heterogeneity, the general solution of the problem should preferable be found with the use of FFT. If, however, the model is asymmetric in terms of the enclosing medium, it would be improper to use FFT for calculations, since the anomalous fields do not belong to the L(-oo,oo) space. In such a case, the class of models permitting the use of the Schwartz algorithm is limited to the above listed set of elementary cases.

Consider a table of solutions in the space of Fourier images and respective originals for E-polarization (Table 1) and for H-polarization (Table 2).

Table 1

Model	Direct FT	Original	Domain
1	$\exp(-\lvert\alpha z\rvert)$	$\frac{\lvert z\rvert}{\pi(y^2+z^2)}$	$z\neq 0, y\in\mathbf{R}$
2	$\exp(-\lvert z\rvert\sqrt{\alpha^2+k^2})$	$G(k,y,z):=\frac{k\lvert z\rvert}{\pi}\frac{K_1(k\sqrt{y^2+z^2})}{\sqrt{y^2+z^2}}$	$z\neq 0, y\in\mathbf{R}$
3	$\frac{sh[\alpha(H-\lvert z\rvert)]}{sh(\alpha H)}$	$\frac{1}{2H}\frac{\sin(\frac{\pi z}{H})}{ch(\frac{\pi y}{H})-\cos(\frac{\pi z}{H})}$	$0\leq\lvert z\rvert\leq H, y\in\mathbf{R}$
4	$\frac{sh[\eta(H-z)]}{sh(\eta H)}$	$\sum_{m=1}^{\infty}[G(k,y,2mH+\lvert z\rvert)-G(k,y,2(m+1)H-\lvert z\rvert)]$	$0\leq\lvert z\rvert\leq H, y\in\mathbf{R}$

Here

$$G(k,y,z) := \frac{kz}{\pi}\frac{K_1\left(k\sqrt{y^2+z^2}\right)}{\sqrt{y^2+z^2}}$$, K_1 - Bessel's function.

Table 2

Table 2

Model	Direct FT	Original	Domain
1	$\exp(-\lvert\alpha z\rvert)$	$\frac{\lvert z\rvert}{\pi(y^2+z^2)}$	$z \neq 0, y \in \mathbf{R}$
2	$\exp(-\lvert z\rvert\sqrt{\alpha^2+k^2})$	$\frac{k\lvert z\rvert}{\pi}\frac{K_1(k\sqrt{y^2+z^2})}{\sqrt{y^2+z^2}}$	$z \neq 0, y \in \mathbf{R}$
3	$\frac{ch[\alpha(H-\lvert z\rvert)]}{ch(\alpha H)}$	$\frac{1}{H}\frac{\sin(\frac{\pi z}{2H})ch(\frac{\pi y}{2H})}{ch(\frac{\pi y}{H})-\cos(\frac{\pi z}{H})}$	$0 \le \lvert z\rvert \le H, y \in \mathbf{R}$
4	$\frac{ch[\eta(H-z)]}{ch(\eta H)}$	$\sum_{m=1}^{\infty}[G(k,y,2mH+\lvert z\rvert)+G(k,y,2(m+1)H-\lvert z\rvert)]$	$0 \le \lvert z\rvert \le H, y \in \mathbf{R}$

Determination of Fields in Half-Bands *(Domain 3 and 4).*

The construction of boundary conditions on lateral borders of grid is proceeded on the basis of use of asymptotic differential equations, adduced in works (Weawer J.T., Brewit-Teylor C.R. 1979,. Zhdanov M.S. atal. 1982, Yudin M.N. 1985).
E-polarisation. The equations for various orders of asimptotics are listed in table.
The solutions of initial boundary value problem for equations, listed in above table, are given in the table 4.
The value y_0 corresponds enough to large significances y, from which become fair asymptotic differential equations. The valuations of y_0 are done by special subprogram, under construction approximate solution of two-dimensional problem on basis of solutions of series of 1D problems.
The significances u_0, u'_0 and u''_0 are defined on network function.
The solution of asymptotic differential equations, listed in table, are used for correction of boundary significances on lateral borders of grid pursuant to Schwartz's algorithm.

Table 3

Degree of asiptotic	Equation $z=0$
1	$y\frac{dE_x}{dy}+E_x=f$
2	$\frac{y^2}{2}\frac{d^2E_x}{dy^2}+2y\frac{dE_x}{dy}+E_x=f$
3	$\frac{y^3}{3!}\frac{d^3E_x}{dy^3}3\frac{y^2}{2!}\frac{d^2E_x}{dy^2}+y\frac{dE_x}{dy}+E_x=f$

The solution of asymptotic differential equations, listed in table, are used for correction of boundary significances on lateral borders of grid pursuant to Schwartz's algorithm.

For correction of boundary significances on lateral borders of grid at z>0 the solution of 1D task appropriate left-hand or right normal model is used. The boundary condition at z = 0 (surface of earth) coincides[1] with solution of asymptotic differential equation.

H-polarisation. For correction of boundary significances of anomal magnetic field during accounts are used (at fixed significances of coordinate z) asymptotic formulas, listed in table. In spite of the fact that the asymptotic behaviour of magnetic field has more the difficult nature, it does not considerable errors in results of solution of general problem.

Table 4

Initial Boundary Values	Solution $z = 0, \quad y > y_a$
$E_x\big\rvert_{y=y_0} = u_0$	$E_x = f + \frac{y_0}{y}(u_0 - f)$
$E_x\big\rvert_{y=y_0} = u_0; E'_x\big\rvert_{y=y_0} = u'_0;$	$E_x = f + \frac{y_0}{y}\left[2(u_0 - f) + y_0 u'_0\right] + \frac{y_0^2}{y^2}\left[f - u_0 - y_0 u'_0\right]$
$E_x\big\rvert_{y=y_0} = u_0; E'_x\big\rvert_{y=y_0} = u'_0; E''_x\big\rvert_{y=y_0} = u''$	$E_x = f + C_1\frac{y_0}{y} + C_2\frac{y_0^2}{y^2} + C_3\frac{y_0^3}{y^3}$

$$C_1 = 3(y_0 - f) + 3y_0 u'_0 + 0.5y_0^2 u'_0; \quad C_2 = -3(y_0 - f) + 5y_0 u'_0 + y_0^2 u'_0;$$
$$C_3 = (y_0 - f) + 2y_0 u'_0 + 0.5y_0 u'_0; \quad f := E''_x.$$

Local Decomposition

In the process of problem solution in the grid domain, local decomposition stemming from its digitization, is generated. Solution in this domain is obtained as a result of iterative recomposition of partial problem solutions found for local subdomains patterned as vertical or horizontal bands, each as wide as four times the grid pitch and overlapping one another by two pitches of the grid (Fig. 2.7, 2.8).

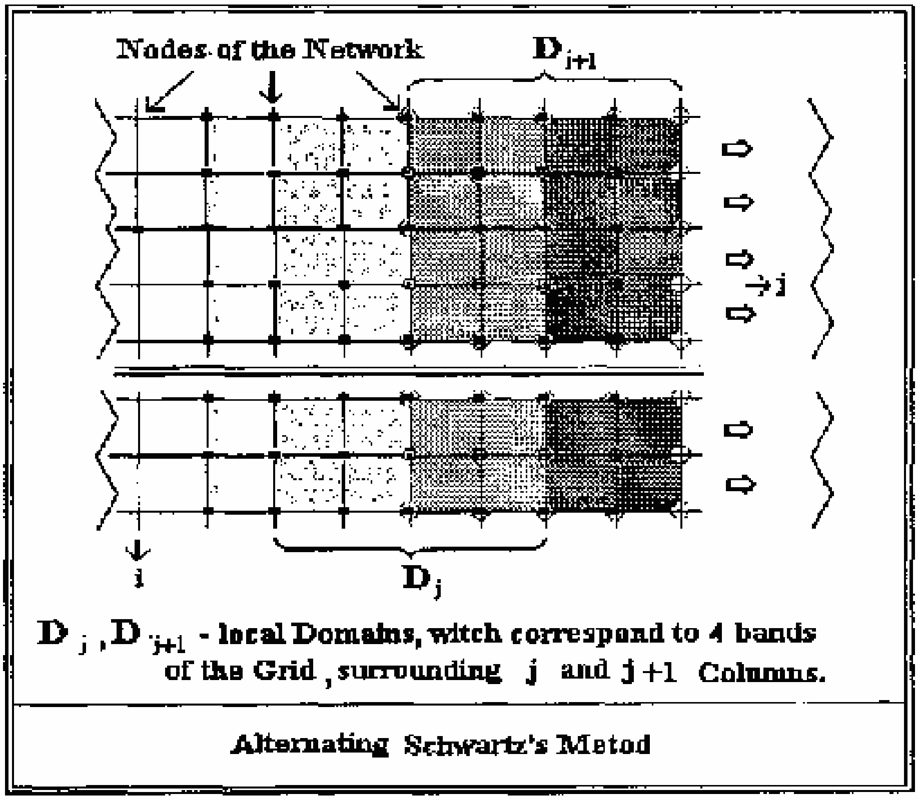


Fig. 2.7

A numerical solution in each of these bands is determined by direct methods, using matrix sweep. The matrix sweep coefficients have the dimension of 3 x 3, therefore any existing inverse matrices have been found analytically. This significantly accelerates computations, as in the

direct methods of solving a set of difference equations, a major portion of computer time is spent for inversion of high-order matrices.
In conformity the Decomposition Altternating Method, let us find a solution for a subset of equations in each block independently and then use the iterative process to sew together the solutions received. This means that a solution must be found in a partial domain of a width equal to twice the grid pitch over the y-axis (Fig. 2.7, 2.8). To find a solution, we have to assume that electric field component values on the right and left boundaries of the domain are known.

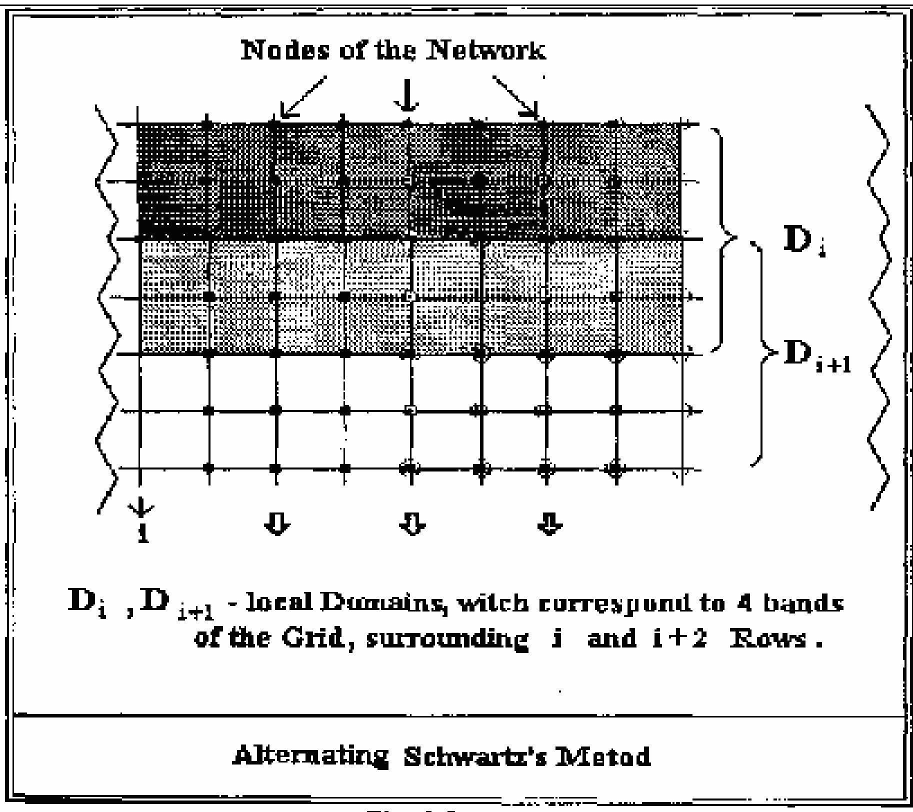


Fig. 2.8.

In conformity with the Decomposition Alternating Method, let us find a solution for a subset of equations in each block independently and then use the iterative process to sew together the 2.3.

The Schwartz Method in 3D Problems

The global decomposition of 3D problems assumes the use of analytical solution three-dimensional boundary problem for top half-space (air) or layer of unlimited thickness, lying in basis of model. The numerical construction of boundary conditions during iterative solution of system of algebraic equations is executed by means of calculation of integrals of convolution on the basis of two-dimensional FFT or direct calculation of double integrals. In the latter case filters use the functions:

$$\psi_1(\mathbf{x},\mathbf{x}_0) = \frac{z}{2\pi R_0^3},\ \sigma = 0,\ \psi_2(\mathbf{x},\mathbf{x}_0) = \frac{z}{2\pi R_0^3}\left(1 + kR_0\right),\ \sigma \neq 0,$$

where k - waver number,

$$\mathbf{x} = (x_1, x_2, x_3),\ \mathbf{x}_0 = (x_1, x_2, x_{3.0}), R_0 = \sqrt{x_1^2 + x_2^2 + (x_3^2 - x_{3.0}^2)}$$

The solution of boundary- value problem in half - space gives the convolution's integral:

$$\mathbf{V}(\mathbf{x}) = \psi_1(\mathbf{x},\mathbf{x}_0) * \mathbf{V}(\mathbf{x}_0)$$

if $V(x_0)$ - boundary values in plane $z = x_{30}$.

3. Algorithms of Numerical Solution of Inner Boundary-Value Problems

Stationary values of the functional are found by the technology of the finite elements method. The software has been designed on the basis of algorithms for rectangular finite elements.

The package includes two groups of programs constructed on the basis of different methods of approximating derivatives of field components that occur in the variation functional.

1. *In the first group,* the derivatives were approximated as finite differences. We shall refer to algorithms of this group as VDM (Variational Difference Methods) ones. Different techniques of approximation resulted in two variants of sets of algebraic equations whose matrices differ in the number of non-zero diagonals. Computation sequences for solving 2D problems have been received as special cases of difference equations of 3D problems with respective assumptions. In 2D problems, the matrix of the set of difference equations appears as a band with five nonzero diagonals. In 3D problems, the number of non-zero diagonals is equal to 13 or 21. Other conditions being the same, the 21-point pattern produces a higher accuracy of calculations than a simples 13-point pattern.
2. *The second group* of difference equations is based on the finite elements method (FEM). Computation sequences were constructed by using linear and exponential test functions which are closely connected with the one-dimensional Green function for a uniform conducting seam. Application of exponential test functions for field approximation over the z-axis provides grounds for receiving an exact numerical solution of MTS problems with any number of dimensions, provided that:
   - exponential approximation is used over z-axis;
   - properties of the medium vary along z-coordinate only; and
   - there exit digitization nodes over z-axis, which coincide with the boundaries of properties discontinuities in the medium model.

3.1. Two-Dimensional Problems in frequency domain

3.1.1. The Finite Elements Method

Проиллюстрируем технологию численного варианта метода Ритца на примере решения двумерной задачи. Для упрощения выкладок примем

- волновое число k – вещественно
- U и V - вещественнозначные функции и
- $U = V \equiv u$.

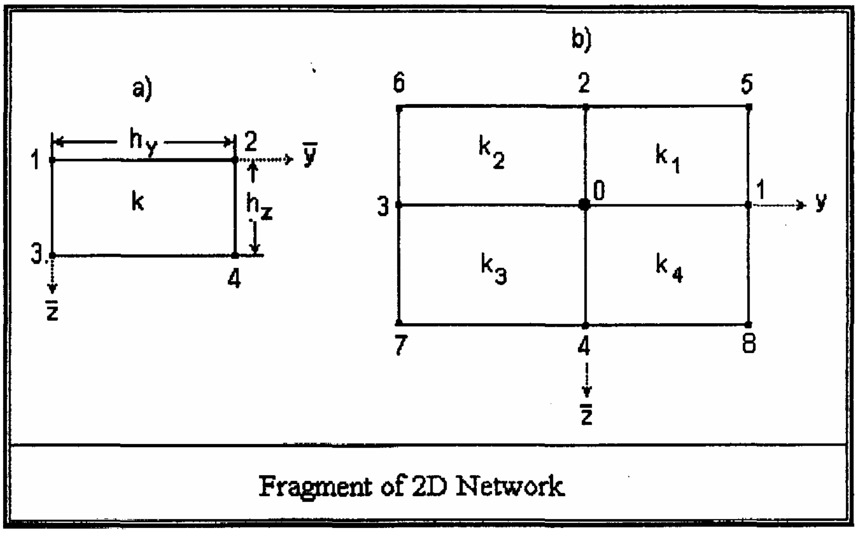
При сделанных предположениях функционал (1.3.6) можно записать в более простом виде

$$\mathrm{F}(u) = \int_D \left\{ \frac{1}{\eta} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 + k^2 u^2 \right] + 2uf \right\} ds,$$

In conformity with the technology of the finite elements method,

$$F(u) = \sum_{i=1}^{N_z} \sum_{j=1}^{N_y} \Delta F_{ij}(u),$$
$$\Delta F_{ij}(u) = \int_{D_{ij}} \left(\left\{ \frac{1}{\eta} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 + k^2 u^2 \right] + 2uf \right\} \right) dydz, \qquad (3.1)$$

where N_{yi} N_z are the numbers of grid nodes along y- and z-axes, respectively; D_{ij} the domain corresponding to one rectangularly shaped finite element (Fig.3.1), (*) is an integrand of the same kind as in Equation (1.4a). Within each element, let us use a local system of coordinates with the origin in left-hand upper comer. Let us introduce a few symbols.



Fragment of 2D Network

Fig.3.1

Let $p_j(y), q_j(z),\ j = 1,2$ denote test functions, such that

$$p_1(0) = p_2(h_y) = q_1(0) = q_2(h_z) = 1,$$
$$p_1(h_y) = p_2(0) = q_1(h_z) = q_2(0) = 0. \tag{3.2}$$

Usually, $p_j(y), q_j(z)$ are linear functions, and in the local system of coordinates they have the form:

$$p_1(y) = 1 - \bar{y}/h_y,\ p_2(\bar{y}) = \bar{y}/h_y,\ \bar{y} = y - y_j$$
$$q_1(z) = 1 - \bar{z}/h_z,\ q_2(\bar{z}) = \bar{z}/h_z, \bar{z} = z - z_m.$$

Here $\bar{y}$ and $\bar{z}$ – local coordinates in a finite element such, that (see fig.3.1a)

$$0 \le \bar{y} = y - y_j \le h_y,\ 0 \le \bar{z} = z - z_j \le h_z.$$

However, when the wave value of wave number k corresponding to the finite element is different from zero, it is appropriate to set

$$q_1(z) = \frac{\mathrm{sh}[k(h_z - \bar{z})]}{\mathrm{sh}(kh_z)},\ q_2(\bar{z}) = \frac{\mathrm{sh}(k\bar{z})}{\mathrm{sh}(kh_z)}$$

The u(y,z) function inside an element may be approximated by the following formula:

$$u(\bar{y},\bar{z}) \cong u_h(x_2,x_3) = \sum_{i=1}^{4} U_i \bar{\tau}_i(x_2,x_3),\ x_2 = y, x_3 = z, \tag{3.3}$$
$$\bar{\tau}_1(x_2,x_3) := p_1(x_2)q_1(x_3),\ \bar{\tau}_2(x_2,x_3) := p_1(x_2)q_2(x_3),$$
$$\bar{\tau}_3(x_2,x_3) := p_2(x_2)q_1(x_3),\ \bar{\tau}_4(x_2,x_3) := p_2(x_2)q_2(x_3)$$

and U_i are the values of the required grid function in the nodes of the finite element. The $\bar{\tau}_i(x_2,x_3)$-function tales the value of 1 in the i-th node and 0 in other nodes.

It is obvious, that

$$u_h^2 = \left(\sum_{i=1}^{4} U_i \bar{\tau}_i\right)^2 = U' T_0 U,$$

if $\hat{T}_0$ - matrix and $U^t \in R^4$ - vector, $\hat{T}_0 = (\tau_i \tau_j)$; i,j = 1,2,3,4 ; $U^t = (U_1,U_2, U_3,U_4)$.
Similarly:

$$\left(\frac{\partial u_n}{\partial y_m}\right)^2 = U^t \hat{T}_m U ,$$

if $\hat{T}_m = \left(\frac{\partial \tau_i}{\partial x_m} \frac{\partial \tau_j}{\partial x_m}\right)$, m = 2,3; i, j = 1,4.

In addition we shall enter a designation

$$T_m := \int_{Dij} \hat{T}_m dx_2 dx_3 = \int_{Dij} \hat{T}_m dydz, m = 0,2,3.$$

then integral on one element $\Delta F_{ij}^h(u_h)$ can be submitted in the following kind:

$$\Delta F_{ij}^h(u_h) = \frac{1}{\eta}\left[U^1(T_2+T_3)U + k^2 U^1 T_0 U\right] - 2U^1 T_0 F , \qquad (3.4)$$

Substituting $u_h(x_2,x_3)$ to right part formula (3.1) after integrating, receive numerical analogue $\Delta F_{ij}^h(u_h)$ of variational functional $\Delta F_{ij}(u)$ by one finite element

$$\Delta F_{ij}^h(u_h) = U^t T U + F^t T_0 U,\ T := \eta^{-1}\left(T_2 + T_3 + k^2 T_0\right),$$

where $F^t = (F_1, F_2, F_3, F_4)$, T, T_0 - stiffness matrixes. As is evident, that
Let

$$P_i := \int_0^{h_y} p_i^2(y)\,dy,\ P_i' := \int_0^{h_y} (p_i')^2\,dy,\ i = 1,2,\ P_{12} := \int_0^{h_y} p_1(y)p_2(y)\,dy,\ P_{12}' := \int_0^{h_y} p_1'(y)p_2'(y)\,dy,$$

$$Q_i := \int_0^{h_y} q_i^2(z)\,dz,\ Q'_i := \int_0^{h_y} (q_i')^2\,dy,\ i = 1,2,\ Q_{12} := \int_0^{h_y} q_1(z)q_2(z)\,dz,\ P_{12}' := \int_0^{h_y} q_1'(z)q_2'(z)\,dz .$$

Assuming that $p_i(y)$ are linear and calculating the integral, we receive:
$P_i = h_y/3,\ P_{12} = h_y/6,\ P_i' = 1/h_y,\ P_{12}' = -1/h_y,$

For exponential basis functions, similar relations have a more complicated form:

$$Q_1 = Q_2 = \frac{h_z}{2sh^2(\eta h_z)}\left[\frac{sh(2\eta h_z)}{kh_z} - 1\right],$$

$$Q_1' = Q_2' = \frac{\eta}{2sh^2(\eta h_z)}\left[h_z\eta + \frac{sh(2\eta h_z)}{2}\right],$$

$$Q_{12} = \frac{1}{\eta 2sh^2(\eta h_z)}\left[h_z\eta ch(\eta h_z) - sh(\eta h_z)\right],$$

$$Q_{12}' = \frac{\eta}{2sh^2(2\eta h_z)}\left[h_z\eta ch(\eta h_z) - 3sh(\eta h_z)\right].$$

Even though integrals of exponential basis functions have a more complicated form, expressions for coefficients of the set of linear relations are obtained from simple relations, which is due to the following identities:

$$Q_1' + k^2 Q_1 = \eta\, cth(\eta h_z),\ Q_{12}' + k^2 Q_{12} = -\eta / sh(\eta h_z).$$

The matrix T will have the following form:

$$T=\begin{Bmatrix} t_0 & t_1 & t_2 & t_3 \\ t_1 & t_0 & t_3 & t_2 \\ t_2 & t_3 & t_0 & t_1 \\ t_3 & t_2 & t_1 & t_0 \end{Bmatrix} \quad T_0=\begin{Bmatrix} a_0 & a_1 & a_2 & a_3 \\ a_1 & a_0 & a_3 & a_2 \\ a_2 & a_3 & a_0 & a_1 \\ a_3 & a_2 & a_1 & a_0 \end{Bmatrix}$$

where

$$\begin{aligned} t_0 &= \eta^{-1}\left(P_1Q_1' + P_1'Q_1 + k^2P_1Q_1\right),\ a_0 = P_1Q_1; \\ t_1 &= \eta^{-1}\left(P_{12}Q_1' + P_{12}'Q_1 + k^2P_{12}Q_1\right),\ a_1 = P_{12}Q_1; \\ t_2 &= \eta^{-1}\left(P_1Q_{12}' + P_1'Q_{12} + k^2P_1Q_{12}\right),\ a_2 = P_1Q_{12}; \\ t_3 &= \eta^{-1}\left(P_{12}Q_{12}' + P_{12}'Q_{12} + k^2P_{12}Q_{12}\right),\ a_3 = P_{12}Q_{12}. \end{aligned} \tag{3.6}$$

It is easy to verify that

$$\frac{\partial}{\partial U_i}\left(U^1 T_m U\right) = 2I^i\left(i\right)TU_m, m = 0,2,3. \tag{3.7}$$

$$\frac{\partial}{\partial U_i}\left(U^i T_m F\right) = I^i\left(i\right)T_m F.$$

where I(i) is a vector whose *i*-th component is equal to 1, while the other components are equal to zero.

In conformity with (3.4) the expression for the derivative will take the form:

$$\frac{\partial \Delta F_{ij}^h}{\partial u_1} = 2\eta^{-1}I^t\left(1\right)TU - 2I^t\left(1\right)T_0F. \tag{3.8}$$

Consider now a fragment of the grid, consisting of four elements with a common node 1 (Fig. 3.1). By summing up the integrals over the four elements, finding a derivative of the variable и and setting it equal to zero, we shall obtain a difference equation associated with the grid node in question. In particular, the equation related to U will be the sum of four expressions of the form (3.8):

$$\sum_{i=0}^{8} C_i U_i = \sum_{i=0}^{8} d_i f_i \tag{3.9}$$

where

$$C_0 = \sum_{j=1}^{4} t_{0,j},\ C_1 = t_{1,1} + t_{1,4},\ C_2 = t_{2,1} + t_{2,2},\ C_3 = t_{1,2} + t_{1,3},\ C_m = t_{4,m}, m = 5,6,7,8.$$

The second subscript in the latter equations indicated the element number to witch they correspond in fig.3.1. The d_i coefficients are determinated using the same relations as for C, but changing t with respective expressions *a* from (3.6).

"Portrait" of System of difference Equations

The boundary conditions are contained in the right-hand part of set (3.10). However, border values cannot be accurately specified before calculations are started. An algorithm on the basis of which these functions can be constructed simultaneously with computation of the numerical solution is discussed in the section 2.

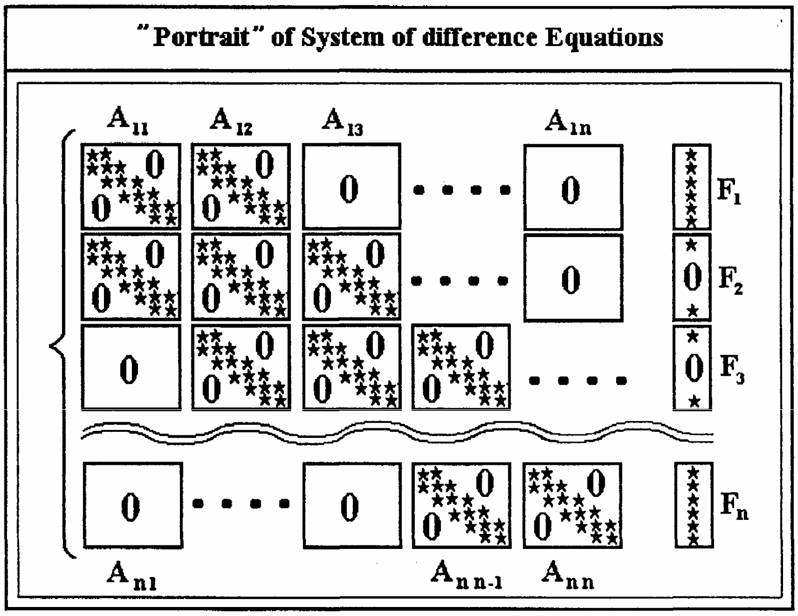


Fig. 3.2. Illustration of the common image of the system of linear equations corresponding to differential task.

3.2. Three-Dimensional Problems

A numerical solution will be found for the following problem in the domain $\Omega \subset R^3$:

$$\begin{cases} F(\mathbf{U},\mathbf{V}) = \langle \eta^{-1} rot\mathbf{V}, rot\mathbf{U} \rangle + \langle k^2 \eta^{-1} \mathbf{V}, \mathbf{U} \rangle - \langle \mathbf{V}, \tilde{\mathbf{f}} \rangle - \langle \mathbf{f}, \mathbf{U} \rangle \\ \mathbf{U}|_{\partial\Omega} = \varphi(P),\ P \in \partial\Omega \end{cases}$$

As the computer has but a limited memory volume, we shall use only those finite elements that are shaped as rectangular parallelepipeds in a rectangular Cartesian system of coordinates. They result from sectioning the 3D space by a set of planes passing parallel to the coordinate planes *OX, OY, OZ.*

Within the limits of each element we shall use local coordinates. The beginning of coordinates corresponds(meets) to near left top top of a cell. Calculations we shall conduct on a grid: The grid will be defined as:

$$\overline{\omega} = \left\{ \left(x_{1i}, x_{2j}, x_{3k} \right) \mid i = \overline{1, N_1},\quad j = \overline{1, N_2},\quad k = \overline{1, N_3} \right\},$$

with steps $h_{mi} = x_{mi+1} - x_{mi}$, m = 1,2,3, $i = \overline{1, N_1 - 1}$.

Let us use local coordinates $\overline{x}, \overline{y}, \overline{z}$ within each element. Coordinates will originate from the nearest left-hand upper node of the cell.

To solve the problem in each of the *M* finite elements, the following set of basic (test) functions will be used:

$$p_{mj}^{(i)}\left(\overline{x}_i\right),\ i = 1,2,3;\ m = 1,2;\ j = \overline{1,M}.$$

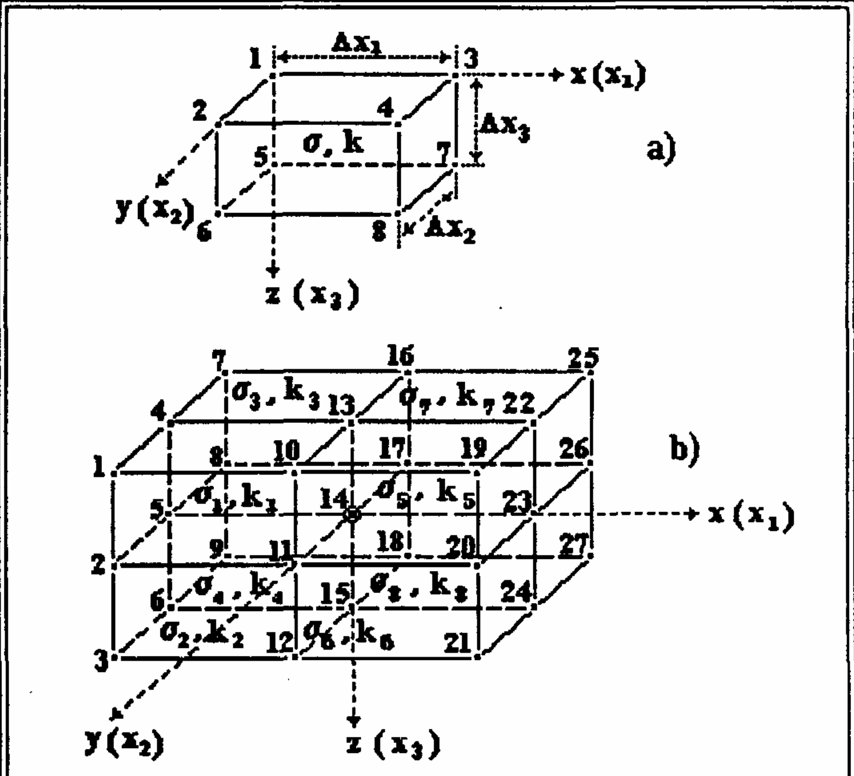
The vector field *U* within each element will be approximated by the relation:

$$\overline{u}_h\left(\overline{x}, \overline{y}, \overline{z}\right) = \sum_{i=1}^{8} \overline{U}_i \tau_i,$$

where each of the functions τ_i is a product of the three basic functions; note that τ_i = 1 in the *i*-th node, while it is equal to zero in all the other nodes. Let's accept

$$\tau_2(\bar{x}_1,\bar{x}_2,\bar{x}_3) = p_1(\bar{x}_1)p_2(\bar{x}_2)p_1(\bar{x}_3),$$
$$\tau_3(\bar{x}_1,\bar{x}_2,\bar{x}_3) = p_2(\bar{x}_1)p_1(\bar{x}_2)p_1(\bar{x}_3),$$
$$\tau_4(\bar{x}_1,\bar{x}_2,\bar{x}_3) = p_2(\bar{x}_1)p_2(\bar{x}_2)p_1(\bar{x}_3),$$
$$\tau_5(\bar{x}_1,\bar{x}_2,\bar{x}_3) = p_1(\bar{x}_1)p_1(\bar{x}_2)p_2(\bar{x}_3),$$
$$\tau_6(\bar{x}_1,\bar{x}_2,\bar{x}_3) = p_1(\bar{x}_1)p_2(\bar{x}_2)p_2(\bar{x}_3),$$
$$\tau_7(\bar{x}_1,\bar{x}_2,\bar{x}_3) = p_2(\bar{x}_1)p_1(\bar{x}_2)p_2(\bar{x}_3),$$
$$\tau_8(\bar{x}_1,\bar{x}_2,\bar{x}_3) = p_2(\bar{x}_1)p_2(\bar{x}_2)p_2(\bar{x}_3).$$

Computation of integrals in all finite elements produces the grid analogue F_h (U, V) of the functional (1.3.2), which is a function of the desired values of the field in grid nodes.
To produce a set of algebraic equations takes finding of partial derivatives of functional F_h for unknown field values in grid nodes, and equate them to zero.
Consider grid fragment shown in Fig. 3.3.



FEM. Fragment of the 3D Network

Fig.3.3. FEM. Fragment of the 3D Network

Each inner node is the common apex of eight elements. A general form of the set of difference equations results from summation of contributions of the eight cells adjoining the node under consideration (№14 in the fig.3.3b).
Each inner *(i,j,k)*- th node of the grid may be described by the following set of three equations:

$$\begin{cases} \sum_{i=1}^{27}\left(U_iC_i^{(1)} + V_iD_i^{(1)} + W_iG_i^{(1)}\right) - R_i^{(1)}F_i^{(1)} = 0, \\ \sum_{i=1}^{27}\left(U_iC_i^{(2)} + W_iD_i^{(2)} + U_iG_i^{(2)}\right) - R_i^{(2)}F_i^{(2)} = 0, \\ \sum_{i=1}^{27}\left(W_iC_i^{(3)} + U_iD_i^{(3)} + V_iG_i^{(3)}\right) - R_i^{(3)}F_i^{(3)} = 0. \end{cases}$$

where C , D, G, R - are coefficients that depend on the electromagnetic properties and the pitch of the grid, and F/ are values of the vector of source densities in digitization nodes.

The total number of such systems is equal to (N_x -2) x (N_y-2) x (N_z-2).

The 3D computational sequences were basic for derivation of equations for 2D problem solution. To achieve this, it must be assumed that only one vector U component (for example, U = (u,0,0)) is different from zero and that its functions depend on two variables (for example, *z* and y), while the medium properties remain constant along the third variable and that the grid pitch is constant. As a result of these assumptions, there appears a common factor of the coefficients, and after cancelling it we receive expressions for coefficients that can be used for the solution of 2D problems.

For reception more difference outlines simple in comparison with finite element method (FEM) private derivative under sign of integral from product the component of vectors of electrical or magnetic fields shall approximate be by finite differences. Using various quadratures we can receive the difference outlines with different orders of approximation derivative ones. The further outline of construction of system of difference equations completely coincides with classical finite elements method (Ritz's method).

FEM in each equation uses the significances of network function in all units of three-dimensional grids, contiguous to fixed internal units (27 points in a pattern). At combination approach to construction of numerical analogue of variational functional the quantity unknown, entering in each equation of system of linear algebraic equations, it is possible to regulate.

Software realizes the algorithm, based on use 21-dot templates. The factors of system of equations are calculated on considerably more to simple formulas, than in FEM.

4. Solution of Auxiliary Problems

4.1. The valuation of sizes of space domain

The model of medium name *locally-normal* in some fixed point in homogeneous two-dimensional or three-dimensional model of medum, if the change of electromagnetic properties on z-axis coincides with change of these properties of 1D model. The electrical field, solution receiving as a result of locally-normal tasks in assumptions constancy of magnetic field on surface of earth, name *agreed* with solution of direct problem for considered 2D or 30 model of medium.

The decomposal approach to solution of direct problem removes the problem of valuation of istances to top and bottom borders of grid. The use of asymptotic boundary conditions on lateral borders of grid requires the receiption of a priori valuations of distances of these borders from studied inhomogeneity.

The two-dimensional problems

The E-polarization. In air the behaviour of electric field is satisfies to Laplace equation . To assume, that it is known the solution of task on surface of earth, top half-space in any point the anomalous electric field is possible find as a result calculation of integral

$$E_x^a(y,z) = \frac{z}{\pi}\int_{-\infty}^{\infty} \frac{E_x^a(\eta,0)\,d\eta}{(y-\eta)^2 + z^2}.$$

Thus, if on basis of solution of series of locally-normal tasks we can a priori satisfactorily estimate the behavior of Ex on surface of earth, problem of valuation of distances to border of grid domain in air can be resolved.
In software there are the module, which provides the valuation of grid domain, adapted to model of media, to period T and asymptotic order.
The H-polarization. In case of H-polarization receives the valuations on programs, the same as E-polarization.

The three-dimensional problems.

The 3D model is considered as the ordered set of 2D models (quasi-2D fragments). For each this fragment the valuations of sizes of settlement domain are proceeded and on basis of analysis of these results is built of 3D grid domain.

4.2. Specificalities of Grid Steps Selection Along the z-Axis

The algorithm of problem solution requires, for optimization of calculations, that calculations should be performed on a fixed grid over axes y and *x.* To choose the pitch along the z-axis, one could use the solution of one-dimensional problem of magnetotelluric sounding, because at large distances from the source, the magnetotelluric field behaves similarly to a flat wave field.
The basic idea in grid pitch selection is as follows. At each fixed time, the principal part of energy of electromagnetic field is concentrated in the upper part of the model. Let the lower boundary of this skin layer be applicate Z *(T).* The function Z *(T)* monotonously increases with the growth of T. The starting time *T* of the problem solution may depend on the performance specifications of the equipment used. If such information is not available (or if it is ignored), the period of *Tbeg* = *T* should be chosen to start when the effect of the anomaly-forming process under study becomes noticeable. Thus, for a 1D model time *T* may be chosen on the basis of the equality Z *(Tbeg) = H1* if *H1* is the thickness of the first seam of a horizontally uniform stratified section. Due to the monotonous character of *Zs (t),* the inverse function exists, therefore:

$$T,=T_{bef}=Z;^{1}(H_l).$$

Proceeding from similar considerations, on the basis of analysis of a 1 D-problem, we can find *T = Zs-1(H).* The grid pitches in each seam are chosen as a certain function of 77 , *pi* and *Hi.*
The idea of algorithm for choosing grid pitches along the z-axis makes it possible to adapt their values to the medium model and to the period at which elements of 2D-model start exerting significant influence upon electromagnetic field.

4.3. Specificalities of Grid Steps Selection Along y-Axis

E-Polarization. Grid selection along y-axis consists of the following steps:
a) Assuming that magnetic field on the ground surface is constant, calculate step function
E"(y) of locally-normal electrical field. To do so, solve the necessary number of one-dimensional problems.
b) The function *E" (y)* is analytically extended into the air some distance from the ground so as to obtain an approximate solution of *E" (y)* of the 2D problem.
c) Function *E" (y)* is utilized in two directions:
- for setting the size **of** the spatial domain along the y-axis and
- for setting grid pitch along this axis.

4.4. Numerical Solution Approximation and Determination of Derivatives of Grid Function

The problem of finding a derivative with respect to an approximately set function (be it a function of a continual argument or a grid function) does not correct stability.
When derivatives are determined for a grid function, the largest errors appear in x-axis derivatives, therefore they deserve a more detailed consideration.
a) As has already been mentioned, in the frequency domain the best approximation of a flat field in a 1D medium within a seam with a wave number *k* and thickness *H* is achieved with functions $q_1(z)$, $q_2(z)$. The approximating function *U(z)* has the following form in this seam:

$$U(z)=U_{i-1}q_1(z)+U_i q_2(z) \qquad (*)$$

where

$$q_1:=\frac{\operatorname{sh}[k(H-h)]}{\operatorname{sh} kH}\exp[-k(H-h)],$$
$$q_2:=\frac{\operatorname{sh} kh}{\operatorname{sh} kH}\exp[-k(H-h)].$$

On the basis of (*), the derivative with respect to *z* can be found as:

$$U'(z)=U_{i-1}q_1'(z)+U_i q_2'(z).$$

b) Determination of derivatives reduced to determination of improper integrals. When converting electrical/magnetic fields into magnetic/electrical, derivatives are determined with the use of Maxwell equations. For the sake of definiteness, we shall consider that numerical solution has produced the values of vector E components in digitalization nodes and that it is the magnetic field that is to be determined.
In the 3D case, horizontal components of vector H can be determined as:

$$H_x(x,y,0)=-\int_0^\infty\left[\sigma E_y+\frac{1}{i\omega\mu}\frac{\partial}{\partial x}\left(\frac{\partial E_x}{\partial y}-\frac{\partial E_y}{\partial x}\right)\right]dz\,,$$
$$H_y(x,y,0)=-\int_0^\infty\left[\sigma E_x+\frac{1}{i\omega\mu}\frac{\partial}{\partial y}\left(\frac{\partial E_x}{\partial y}-\frac{\partial E_y}{\partial x}\right)\right]dz\,.$$

E-Polarization. the following integral is an analogue to the latter equations:

$$H_y(x,y,0)=-\int_0^\infty\left[\sigma E_x+\frac{1}{i\omega\mu}\frac{\partial^2 E_x}{\partial y^2}\right]dz\,.$$

c) Derivatives determined with the use of Fourier transform. Consider an algorithm for determination of derivatives in the case of E-polarisation. Proceeding from Maxwell equations, it is easy to obtain the following forms of equations for determination of components *Hy* and *Hz* on the ground surface (E-polarisation):

$$H_y(y,0)=\frac{1}{i\omega\mu}F^{-1}\left\{\alpha F\left[\sigma E_x^a(y,0)\right]\right\}+H_y^n(0)$$
$$H_z(y,0)=\frac{1}{i\omega\mu}F^{-1}\left\{i\alpha F\left[E_x^a(y,0)\right]\right\}+H_z^n(0)$$

and

$$E_y(y,0)=\frac{1}{\sigma}F^{-1}\left\{-\sqrt{\alpha^2+k^2}F\left[E_x^a(y,0)\right]\right\}+E_y^n(0)$$

in H-polarisation case.
Similar formulas for transformation of E vectors into H ones (or H into E) exist for 3D fields.

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Appendix 1. The list of conditional designations and reductions

E - electric field
H - magnetic field
$\mathbf{H}^a$, $\mathbf{E}^a$ - anomalous electromagnetic field
$\mathbf{H}_n$, $\mathbf{E}_n$ - normal electromagnetic field
j - density of electric current
J - force of current
I - moment of electrical dipole
M - moment of magnetic dipole
μ - magnetic permeability
μ_0 - magnetic permeability of air
σ - specific electric conductivity
ρ - specific electric resistance
ε- dielectric permittivity
k - wave number
λ - length of electromagnetic wave
ω - circular frequency
T - period of fluctuations
n-normal unit vector
τ- tangential unit vector
H_τ E_τ - tangential components of vectors **H** and **E**
U, $\mathbf{U}_h$ - vector - function **U** and its network analogue $\mathbf{U}_h$
[f]| -jump of function f on border of domain
K_n (z) - Macdonald's function of order n
u, u* - complex conjugate functions

Appendix 2. Simple example.

To evaluate the speed of convergence of Schwartz method it is possible on elementary task, possessing the analytical solution [2].
Let is required on algorithm of Schwartz to decide the boundary-value problem on hatf-direct z > 0 at constant significance of wave number k:
dz^2

$$\begin{cases} \dfrac{d^2\,\mathrm{U}}{d\,z^2} = k^2\,\mathrm{U}\,, \quad \mathrm{U}_{z=0} = 1\,; \quad \mathrm{U} \to 0, z \to +\infty\,, \\ k = const. \end{cases}$$

The solution of task (3.1) obviously of: u (z) = exp (-kz). In consent with algorithm consistently decide two tasks

$$\begin{cases} 1.\quad \dfrac{d^2\,\mathrm{u}^{(2m-1)}}{d\,z^2} = k^2\,\mathrm{u}^{(2m-1)}, \qquad \mathrm{u}^{(2m-1)}\Big|_{z=0} = 1 \\ z \in (0,H); \qquad \mathrm{u}^{(2m-1)}\Big|_{z=H} = \mathrm{u}^{(2m-2)}(H); \quad \mathrm{u}^{(0)} = a\,; \\ \qquad\qquad m = 1,2,3\ldots \\ 2.\quad \dfrac{d^2\,\mathrm{u}^{(2m)}}{d\,z^2} = k^2\,\mathrm{u}^{(2m)}, \qquad \mathrm{u}^{(2m)}\Big|_{z=h} = \mathrm{u}^{(2m-1)}(h)\,; \\ z \in (h,\infty); \quad h \langle\, H\,; \qquad \mathrm{u}^{(2m-1)} \to 0 \quad npu \quad z \to \infty \end{cases}$$

The significance of constant a is given arbitrarily.

$$u^{(1)}(z) = \frac{\text{sh}\left[k(H-z)\right]}{\text{sh}\, kH} + a\frac{\text{sh}\, kz}{\text{sh}\, kh},$$

$$u^{(2)}(z) = u^{(1)}(h)\exp\left[-k(z-h)\right],$$

Therefore, in particular, at z = H we receive

$$u^{(2)}(H) = u^{(1)}(h)\exp\left[-k(H-h)\right] = \frac{\text{sh}\left[k(H-h)\right]}{\text{sh}\, kH}\exp\left[-k(H-h)\right] + a\frac{\text{sh}\, kz}{\text{sh}\, kh}\exp\left[-k(H-h)\right].$$

.

Assume

$$q_1 := \frac{\text{sh}\left[k(H-h)\right]}{\text{sh}\, kH}\exp\left[-k(H-h)\right],$$

$$q_2 := \frac{\text{sh}\, kh}{\text{sh}\, kH}\exp\left[-k(H-h)\right].$$

It is possible to record recurrent formula

$$u^{(2m)}(H) = q_1 + u^{(2m-2)}\, q_2 .$$

Whence follows

$$u^{(2m)}(H) = q_1\sum_{l=1}^{m} q_2{}^{l} + u^{(0)}\, q_2{}^{m+1} = q_1\frac{1-q_2^m}{1-q_1} + u^{(0)}\, q_2{}^{m+1}$$

It is obviously, that | q_i || < 1, therefore

$$\lim_{m\to\infty} u^{(2m)}(H) = \frac{q_1}{1-q_2}$$

conterminous with solution of task (3.1) at z = H. Hence, at any significance z

$$\lim_{m\to\infty} u^{(2m)}(z) = u(z) = \exp(-kz).$$

Thus, on method of Schwartz we can receive the solution of problem (3.1) at any initial approach to boundary significance 3,

The convergence of iterative process will the theme faster, than less the size q and than closer the initial approach a to exact boundary significance. In particular, assuming a = 1, h = H / 2, | kH| =1, error of solution will be less then 1 % afterwards 5 iterations.

The approach h to H lead to increase q and deterioration of convergence of iterative process to solution. So, at h / H = 0.9 for achievement the same accuracy 18 iterations is required.

The speed of convergence depends also from size of conductivity of media (and, hence, size of wave number *k*). As results from (3.4), at k -> 0, q -> h / H, that can serve by estimate from above for size of this parameter in conducting environment media.